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Algorithmic Trading Division

Trading VIX Futures via Volatility Risk Premium Estimation using GARCH and HAR-RV models

Vienna, 03/02/2022

Team Overview

Algorithmic Trading

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Head of Algorithmic Trading

- Concept
- Backtesting

- MSc. – earned
- BSc. – earned

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- HAR RV modelling
- Backtesting

- MSc. – 3rd sem.
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- Concept

- BSc. – earned

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- GARCH modelling

- MSc. – 5th sem.
- BSc. – earned

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Introduction to Volatility Trading

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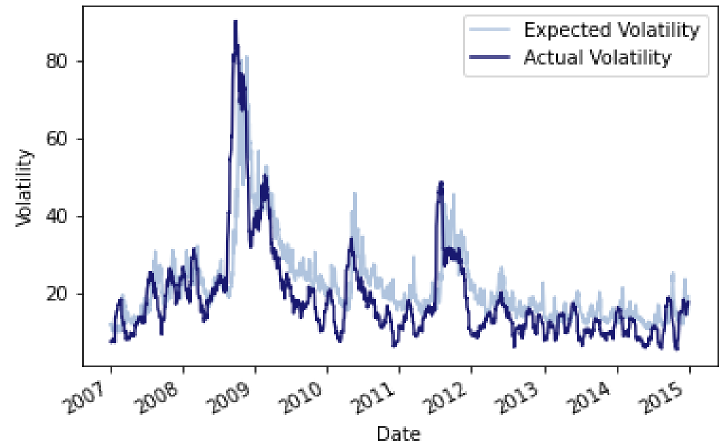
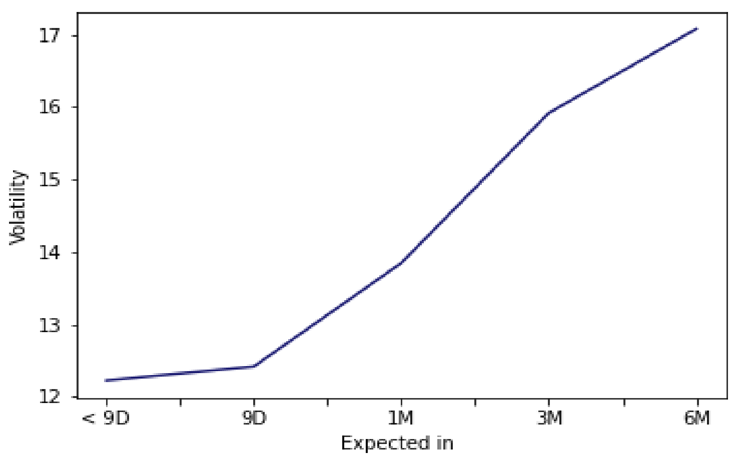
The Basics

- Volatility is the degree of variation of returns for a given asset
- Buying volatility is the same as buying an insurance for a given asset
- Volatility risk premium is a phenomenon that option-implied volatility tends to exceed actual volatility
- There are numerous ways to estimate the VRP, we are interested in two:
  - Using the expected (implied) term structure
  - Using actual (realized) volatility forecasts

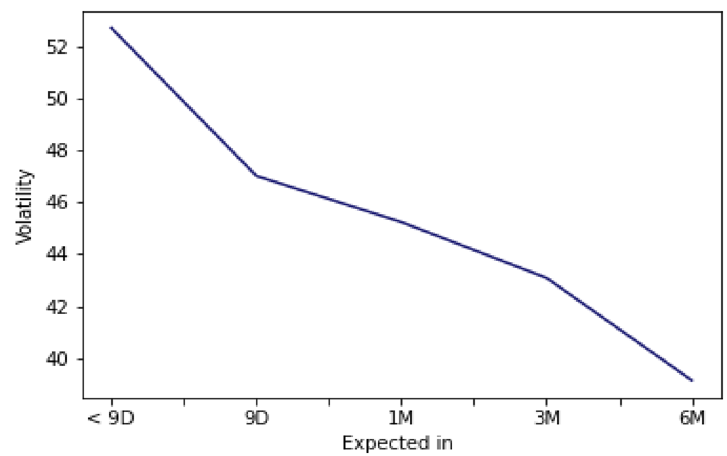
Actual vs Expected Volatility



Implied Volatility Surface on 06/01/2020 (pre COVID crash)



Implied Volatility Surface on 06/04/2020 (during COVID crash)



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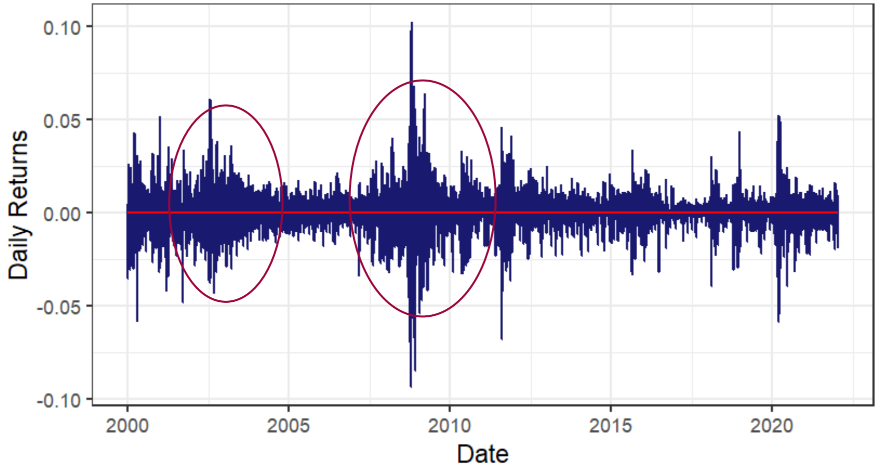
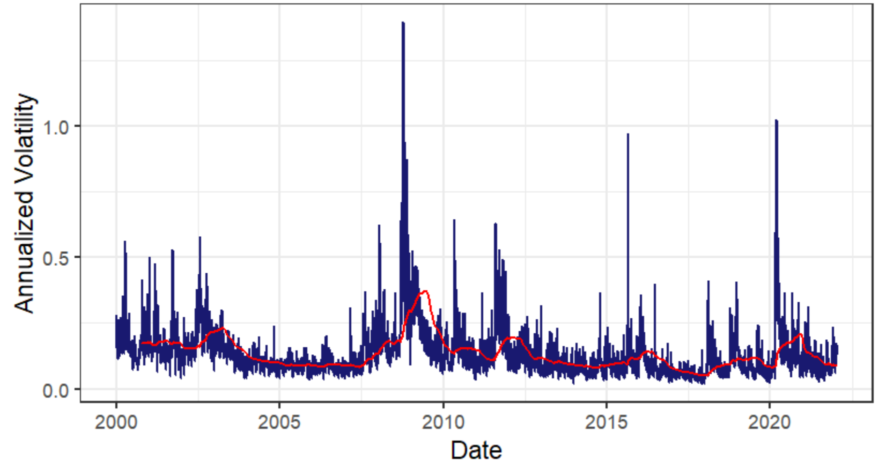
Volatility properties and GARCH modelling

Volatility Clustering



Mean Reversion



GARCH models

Models:

- GARCH: $\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$, where $\sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j < 1$
- IGARCH: $\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$, where $\sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j = 1$
- GJR-GARCH: $\sigma_t^2 = \omega + \sum_{j=1}^{q} (\alpha_j \varepsilon_{t-j}^2 + \gamma_j I_{t-j} \varepsilon_{t-j}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$
- EGARCH: $\log \sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j g(Z_{t-j}) + \sum_{j=1}^{p} \beta_j \log \sigma_{t-j}^2$
- Asymmetric Power GARCH: $\sigma_t^\delta = \omega + \sum_{j=1}^{q} \alpha_j \left(|\varepsilon_{t-j}| - \gamma_t \varepsilon_{t-j} \right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta$

GARCH models (cont.)

- Component GARCH: $\sigma_t^2 = q_t + \sum_{j=1}^{q} \alpha_j (\varepsilon_{t-j}^2 - q_{t-j}) + \sum_{j=1}^{p} \beta_j (\sigma_{t-j}^2 - q_{t-j})$
- Realized GARCH: $\log \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \log r_{t-i} + \sum_{i=1}^{p} \beta_j \log \sigma_{t-i}^2$

Distributions:

- Normal distribution
- Skewed normal distribution
- Student's t-distribution
- Skewed Student's t-distribution
- Generalized error (normal) distribution
- Skewed generalized error (normal) distribution
- Normal-inverse Gaussian
- Generalized Hyperbolic

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Volatility Forecasting Performance of GARCH Models

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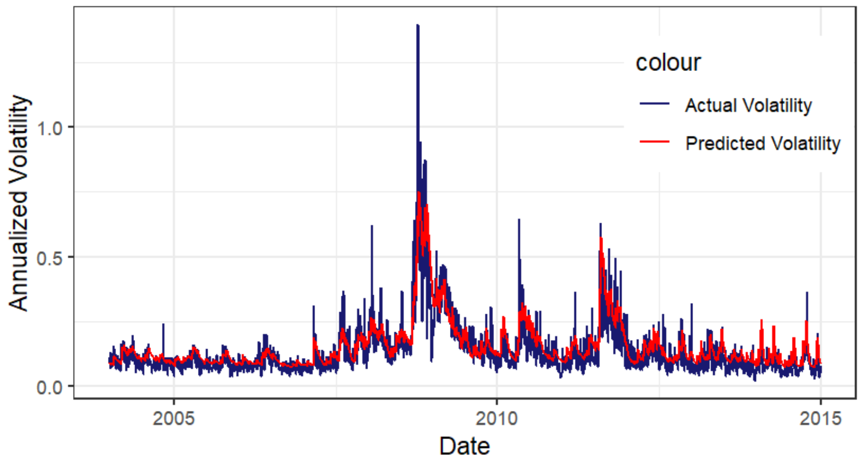
Model specification

- Test period [2004:2014]
- GARCH model
- GARCH (p,q) order
- ARMA (p,q) order
- Error distribution
- Refit window
- Error Measure (QLIKE, MSE)

GARCH(1,1), ARMA(0,0), gjrGARCH, snorm 2 year moving



Volatility forecast loss function

- $MSE = \frac{1}{n}\sum_{i=0}^{n}(y_i - \hat{y}_i)^2$

- $QLIKE = \frac{R_{t+1}^2}{\sigma_{t+1}^2} - \ln\left(\frac{R_{t+1}^2}{\sigma_{t+1}^2}\right) - 1$

Model performance

Refit Window	4Y Exp	4Y Roll	2Y Roll	1Y Roll
	GJR-GARCH(1,1), Skewed Norm.	GJR-GARCH(1,1), Skewed Norm.	GJR-GARCH(1,1), Skewed Norm.	GJR-GARCH(1,1), General. Norm.
QLIKE	0.2245	0.2195	**0.2079**	0.2080
MSE	0.0037	0.0035	**0.0033**	0.0035

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Heterogenous Autoregressive (HAR) Models

High-Frequency-Based Measures for Volatility Modeling

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Preliminaries on Latent Volatility Process Approximation

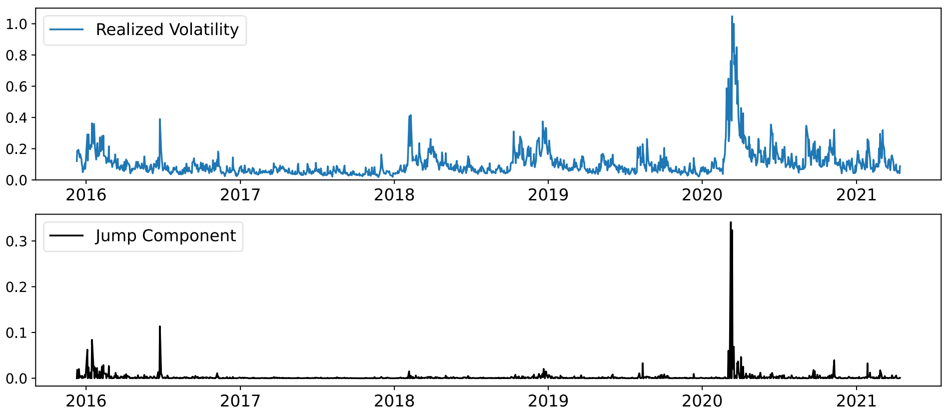
- Noisy observed log-price: $S_t = Y_t + U_t$, where $dY_t = \mu_t dt + \sigma_t dW_t$
- Integrated Volatility (IV): $\int_0^t \sigma_s^2 ds$, can be proxied efficiently from HF data
- Realized measures: tradeoff between sampling frequency and noisiness
- Realized Variance $RV_t := \sum_{\{i=1\}}^{N} R_{t,i}^2$, i.e. sum of squared intraday returns
- Bipower Variation: $BV_t := \frac{N\,\pi}{2(N-1)} \sum_{\{i=2\}}^{N} |R_{t,i}||R_{t,i-1}|$, continuous component
- Jump: $J_t := \max[RV_t - BV_t, 0]$
- Modeled Variable: $RV_t = (RV_t^{5min} + RV_t^{10min})/2 + Overnight\,Variation_t$

HAR-RV vs. HAR-RV-J

- Realized Variance exhibits scaling and multiscaling
- Returns PDFs are leptokurtic; shapes are timescale-dependent
- Standard GARCH & SV models fail to explain above features
- HAR-RV also considers the long memory stylized fact of volatility process
- No information loss due to daily granularity of intraday returns sampling
- HAR-RV: $RV_{\{t,t+1\}} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \varepsilon_{\{t,t+1\}}$
- HAR-RV-**J**: $RV_{\{t,t+1\}} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \boldsymbol{\beta_J J_t} + \varepsilon_{\{t,t+1\}}$

  s.t. RV_t, $RV_{t-5,t}$, and $RV_{t-22,t}$, are daily, weekly, and monthly RVs observed at t

Decoupling Jump Component from Realized Variance



Model Selection and (Out-of-)Sample Performance

	HAR-RV, 2Y Roll	HAR-RV, 4Y Roll	HAR-RV, 6Y Roll	HAR-RV, 4Y Exp
Train, QLIKE	0.2580	0.2556	0.2548	**0.2504**
Train, MSE	0.0042	0.0041	0.0040	**0.0037**
Trading, QLIKE	0.3962	0.5111	0.3657	**0.3549**
Trading, MSE	0.0104	0.0092	0.0072	**0.0051**

	HAR-RV-J, 2Y Roll	HAR-RV-J, 4Y Roll	HAR-RV-J, 6Y Roll	HAR-RV-J, 4Y Exp
Train, QLIKE	0.3163	0.3121	0.3137	**0.2590**
Train, MSE	0.0054	0.0054	0.0051	**0.0035**
Trading, QLIKE	0.4844	0.4401	0.4205	**0.3630**
Trading, MSE	0.0061	0.0059	**0.0056**	0.0061

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Backtest

Methodology and Training Period Performance

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Methodology

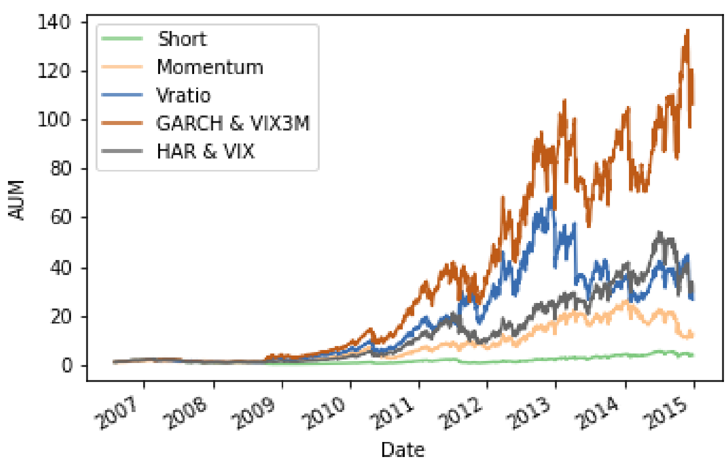
- Instrument traded – SPVXSPID
- Training Period (08/2006 – 12/2015), Test Period (01/2016 – 11/2021)
- Benchmarks
  - Short
  - Momentum
  - Vratio (VIX – VIX3M)
- Our models
  - HAR with expanding optimization window
  - GJR-GARCH with rolling 2-year optimization window
- No trading costs, all models are optimized daily at close, trades are also done daily at close

Performance Overview

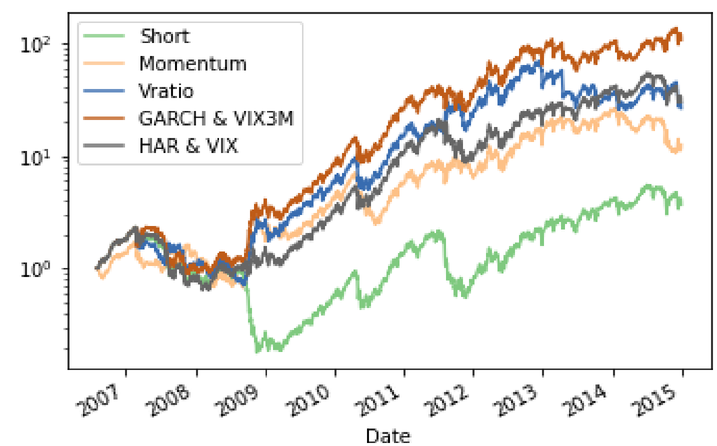
	Annualized Return	Annualized Volatility	Sharpe Ratio
GARCH & VIX3M	74.0%	63.9%	1.14
HAR & VIX	49.7%	63.9%	0.76
Vratio	47.5%	63.9%	0.72
Momentum	35.0%	63.9%	0.53
Short	16.8%	63.9%	0.25

- Could it be that for us VIX3M is a more suitable IVOL index than VIX?
- In the test sample we will use the following IVOL indices:
  - VIX9D
  - VIX3M
  - VIX
  - VIX6M

Linear Scale Performance



Logarithmic Scale Performance



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Test Period Performance and Findings

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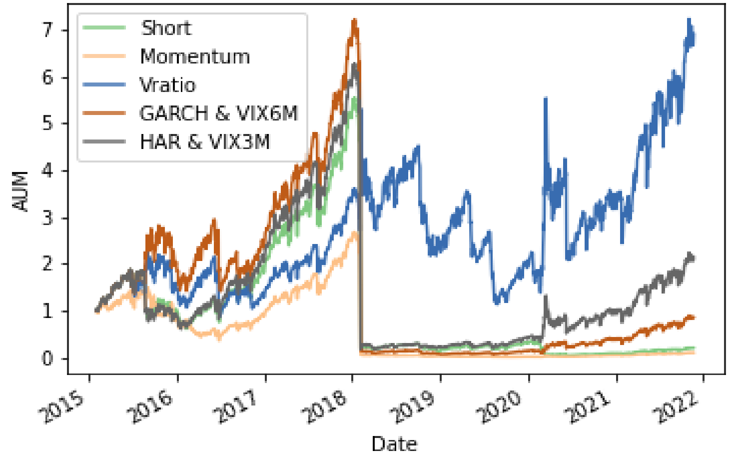
Performance Overview (Full Test Sample)

	Annualized Return	Annualized Volatility	Sharpe Ratio	Return on 05/02/2018
Vratio	32.8%	82.0%	0.39	Positive
HAR & VIX3M	11.8%	82.0%	0.13	Negative
GARCH & VIX6M	-2.1%	82.0%	-0.02	Negative
Short	-20.7%	82.0%	-0.26	Negative
Momentum	-27.8%	82.0%	-0.28	Negative

- 97% of AUM in the backtest was destroyed on one day for most strategies when VIX went from 18.44 to 37.32
- Was the Vratio model simply lucky?

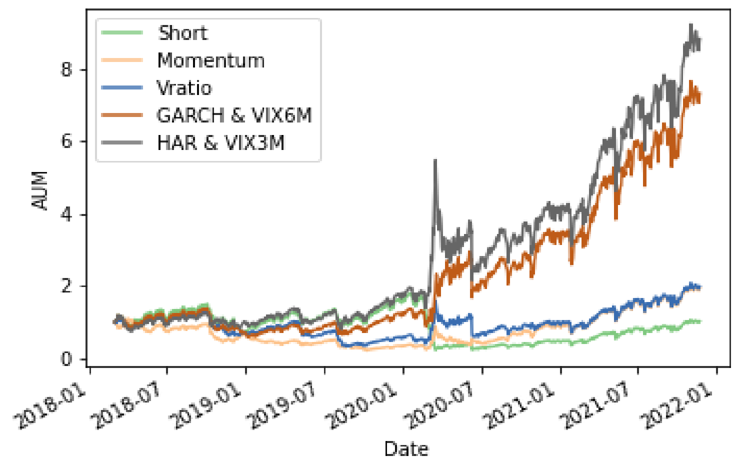
Linear Scale Performance



Performance Overview (03/2018 – 11/2021)

	Annualized Return	Annualized Volatility	Sharpe Ratio
HAR & VIX	80.9%	76.3%	1.03
GARCH & VIX6M	70.0%	76.3%	0.70
Vratio	20.0%	76.3%	0.20
Momentum	19.3%	76.3%	0.19
Short	0.1%	76.3%	-0.01

Linear Scale Performance



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Backtest

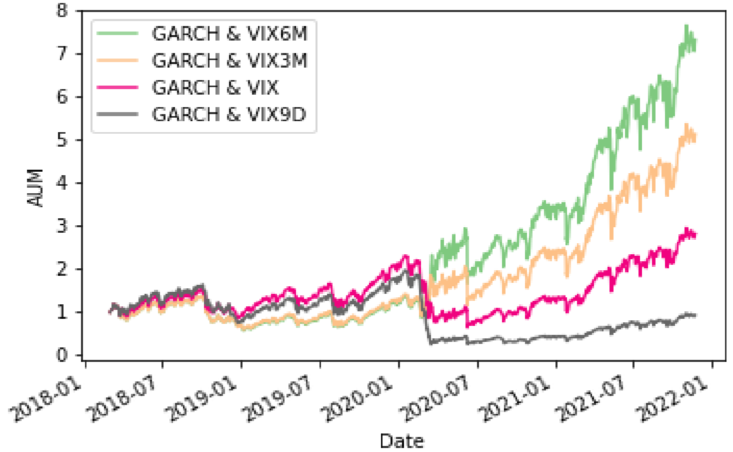
IVOL Measure Selection and Conclusion

IVOL Measure Selection

	Best	2nd	3rd	4th
GARCH	VIX6M	VIX3M	VIX	VIX9D
HAR	VIX	VIX3M	VIX6M	VIX9D

- Somewhat inconclusive evidence, GARCH performance order is what we expected, while the performance order of HAR is not
- For GARCH models the performance during economic downturns heavily depends on the IVOL measure

GARCH Performance Comparison



Conclusions

- Volatility trading is risky (a shocker, we know)
- Performance is best during market downturns
- Both HAR and GARCH models outperform the benchmark (ex. 05/02/2018)
- GARCH models perform best when combined with VIX6M
- Potential further research topics:
  - Trade only when the signal is “buy“?
  - Combine signals from several strategies and trade only when both match?

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